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RESEARCH REPORT

Mass and stiffness data and calculations on blade vibration properties

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Summary

The report contains the results of calculations of mass and stiffness data and calculations of vibration properties of composite blades of a small helicopter.

Table of signs

D	m	rotor diameter
c	m	rotor chord
E	N/mm2	Young's modulus
J	m4	geometrical moment of inertia
x, y, z	m	coordinates in the global system of the rotor axis
F	N	force
M	Nm	torque
α	rad	angle
n	rps	speed of the rotor
g	m/s2	gravity acceleration
CG		centre of gravity
CLF		centre of lateral forces

1. INTRODUCTION AND PURPOSE OF WORK

The report contains basic data and results of mass and stiffness calculations for helicopter composite rotor blades and the results of vibration characteristics calculations.
The calculations were performed to determine the basic blade vibration frequency and to compare the so calculated blade properties with the results of stiffness tests of the actual blade.

The report contains the results of calculations of decomposition of:
- Mas,
- Stiffness,
- CG position,
- CLF position.

The calculation results are intended to verify the calculations for the rotor and its dynamic properties and may be subject to further changes.

2. DETAILS OF THE BLADE

Fig. 1 shows a blade along with its balancing tubes.

Basic rotor dimensions:

Blade length L = 3.25 m
Airfoil chord c = 180 mm
Blade weight M = 8.885 kg
Rotor diameter D = 7.5 m

The blade can be divided into 4 sections:

1. Root section – section covered by the hub bar (fixing), 300 mm length
The section is almost homogeneous: 5 holes for fixing bolts, at the distance 100 mm is hole for the balancing weight, a formed transition to the blade profile,

2. The inner section - 1870 mm length
without balancing elements, homogeneous section

3. The outside section - 980 mm length
one tube for balancing rod Ø 10 mm, homogeneous section

4. The end part – 100 mm length
two tubes for balancing rods Ø 10 and Ø 12 mm, homogeneous section

Some elements have been removed from the report

since they contain secret company's data

Fig. 1 A blade sketch

3. STIFFNESS-MASS DISTRIBUTION

Fig. 2 presents a model for FEM calculations for a blade.

The model consists of approx. 34 000 items type "solid".

Defined were 3 basic construction materials of a blade (based on information from the sample):
- Sheathing < *Some elements have been removed from the report since they contain secret company's data*>
- Carbon roving < *Some elements have been removed from the report since they contain secret company's data*>
- Foam < *Some elements have been removed from the report since they contain secret company's data*>

and lead a balancing weight.

The reference mass of the blade:
M = 8.872 kg
including mass m = 0.36 kg in the form of balancing at the blade end.

The calculations assume the origin of the coordinate system at a point on the leading edge on the blade root (see fig. 2). The position of the CG in the adopted calculation model.
X = -0.0556 m Y= -0.0001 m Z = 1.610 m

The position of the CG of the blade provided for measurement: Z = 1763 mm.
The difference in the position the centre of gravity results from unknown quantity and density of the material filling the root section of the blade taken for the calculation: the filling and density taken for the calculation was the same as for roving. This difference has no significant effect on the stiffness properties of the blade apart from the root section (described in Sect. 3.1.).

The moments of inertia of the blades in relation to the origin of the coordinate system:

I_{XX} = 31.557	I_{XY} = 0.00128
I_{YY} = 31.597	I_{YZ} = -0.0139
I_{ZZ} = 0.0410	I_{ZX} = -0.7980

Max (computing) weight of the balancing rods, (assuming material: lead, a rod pressed on epoxy glue)

on the root: Ø 10.0 mm l_1 = 100 mm	m ≈ 0.085 kg
at the end: Ø 12.0 mm l_2 = 1 080 mm	m ≈ 1.380 kg
Ø 10.0 mm l3 = 100 mm	m ≈ 0.085 kg

To determine the mass and stiffness data of individual fragments prepared were homogeneous computational models for FEM length 1m, modeling full restraint at one end.
The data were reported in units: kg and m in a coordinate system of the airfoil (corresponding to the coordinates of the blade system).

CLF calculations were made on the basis of deflection of the extreme points of the profile (on the leading edge and on the trailing edge) giving control loads (cross force) near the CLF point. The calculations were made for small loads in a linear range and CLF location was determined basing on deflection charts for selected points, depending on the position of control power.

Stiffness calculations were performed on the basis of deflection of the extreme points of the profile giving control loads at CLF point. Calculations were made for small loads, in linear range of stress.

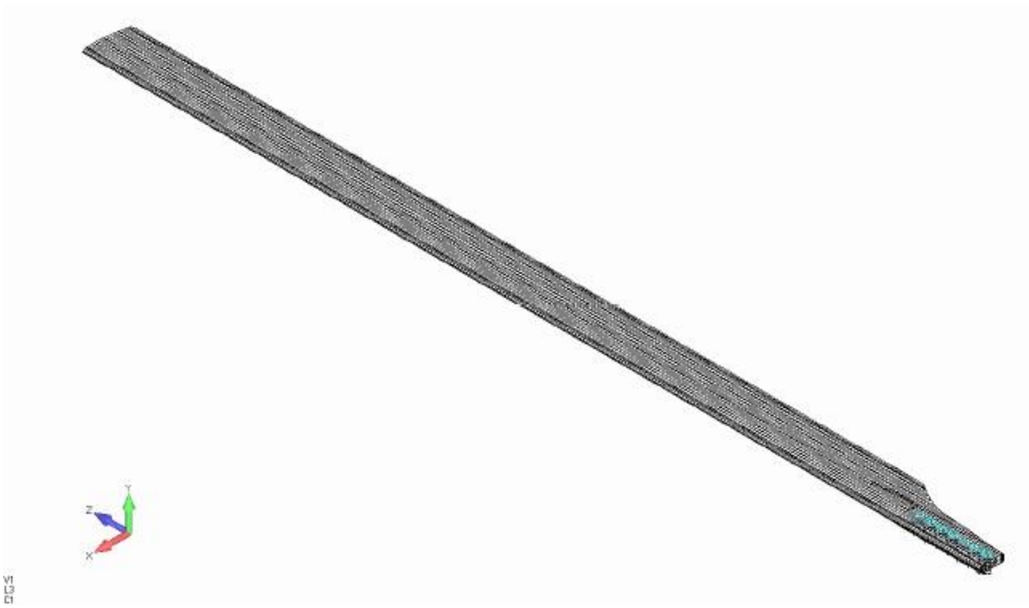

Fig. 2 A blade model taken to FEM calculations

3.1. Root section

Fig. 3 shows the profile of the blade model taken for FEM calculations in the root section, between fixing holes (division into parts).

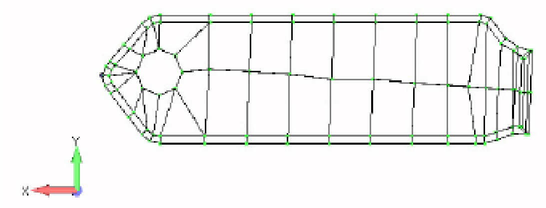

Fig. 3. Part of the blade in root section between holes - model for FEM.

Both profile and its weight change in the section of approx. 0.40 m.
Because of the relatively high stiffness of the hub bar (blade attachment element) and the blade (bold in this section), CG calculations, CLF calculations and stiffness calculations were not performed.
For the profile presented in the Fig. 3 the CG is at the point of 0.047 m from the leading edge and weight of a section of length. 0.01 m is 0.69 kg.
For the purposes of calculations for this section, CLF and CG can be assumed at 0.045 m from the leading edge i.e. in the holes plane.

3.2. Middle section

Fig. 4 shows the blade profile in the central part of it, divided into components.

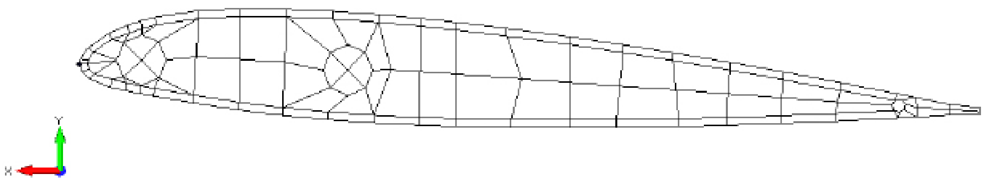

Fig. 4 Middle section of the blade - model for CLF and stiffness location calculation.

Weight of a blade profile with a length of 1 m:

m1 = 2.580 kg

CG location:

X = -0.0580 Y = -0.0011

Moments of inertia towards the coordinate origin:

I_{XX} = 0.8599 I_{XY} = 0.000421

I_{YY} = 0.8726 I_{YZ} = -0.001365

I_{ZZ} = 0.01284 I_{ZX} = -0.07480

Moments of inertia towards CG:

I_{XX} = 0.2150 I_{XY} = 0.0002627

I_{YY} = 0.2190 I_{YZ} = 4.722 E-14

I_{ZZ} = 0.00416 I_{ZX} = 2.607E-13

CLF position:

Deflection calculated in two extreme points of the model profile of the blade (1 m length) under the force F_Y = 100 N (‘pkt’ is a node number in the calculation model).
(N - point on the leading edge of the profile, S - point on the trailing edge of the profile).

X [m]	Y_N [m]	Y_S [m]
	pkt 13039	pkt 212
0.065	0.023792	0.023784
0.070	0.023786	0.023793
0.075	0.023781	0.023802

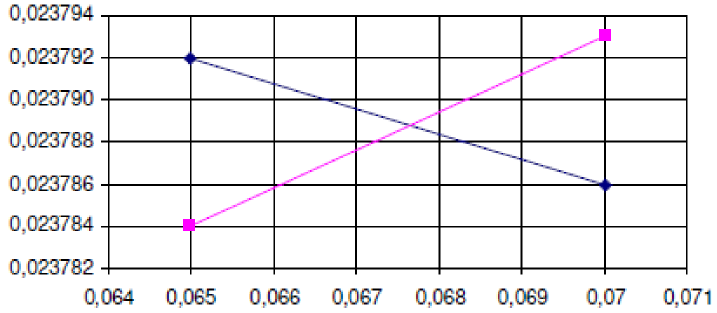

Fig. 5 Chart to determine CLF position

The intersection point corresponds to the position:

$$X_{CLF} = 0.0677 \text{ m}$$

corresponding to 37.6% of the blade chord.

Deflection of a 1 m blade fragment at a torque of M_X = 100 Nm:

Y_N [m]	Y_S [m]
pkt 13039	pkt 212
0.035725	0.035725

Average deflection: y = 0.035725 m

Flexural stiffness in a plane perpendicular to the rotor plane (lower blade stiffness):

$$EJ_X = M_X / (2*z) = 1\,399 \text{ Nm}^2$$

Deflection of a 1 m blade fragment at a torque of M_Y = 100 Nm:

X_N [m]	X_S [m]
pkt 13039	pkt 212
0.000924	0.000924

Average deflection: x = 0.0012235 m

Flexural stiffness in the rotor plane (greater blade stiffness):

$EJ_Y = M_Y / (2*z) = 54\,110 \text{ Nm}^2$

Deformation (twisting) of the 1 m blade section under twisting forces $M_Z = 100$ Nm:

$$\alpha = 0.01167 \text{ rad}$$

Torsional stiffness:

$$GJ_O = MZ/\alpha = 8\ 569 \text{ Nm}^2/\text{rad}$$

3.3 External section

Fig. 6 shows the blade profile in a section with one balancing tube, divided into elements.

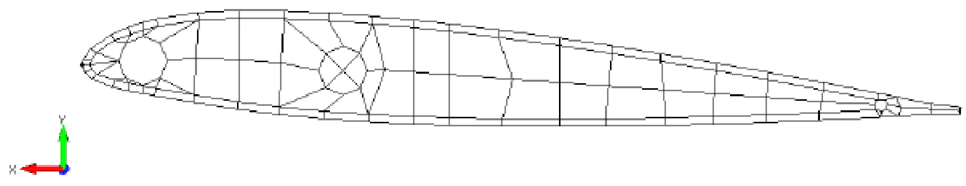

Fig. 6 External section of a blade – model for CLF and stiffness location calculation.

Weight of the 1 m blade profile:

$$m_2 = 2.468 \text{ kg}$$

CG location:

$$X = -0.0621 \quad Y = -0.0013$$

Moments of inertia towards the coordinate origin:

$$I_{XX} = 0.7105 \quad I_{XY} = 0.000408$$
$$I_{YY} = 0.7222 \quad I_{YZ} = -0.01377$$
$$I_{ZZ} = 0.0119 \quad I_{ZX} = -0.06620$$

Moments of inertia towards CG:

$$I_{XX}= 0.1777 \quad I_{XY} = 0.0002373$$
$$I_{YY} = 0.1812 \quad I_{YZ} = -4119 \text{ E-12}$$
$$I_{ZZ} = 0.00368 \quad I_{ZX} = -2.469\text{E-11}$$

CLF position:

Deflection calculated in two extreme points of the model profile of the blade (1 m length) under the force $F_Y = 100$ N

X [m]	Y_N [m]	Y_S [m]
	pkt 12952	pkt 437
0,06	0,023809	0,023786
0,065	0,023803	0,023795
0,070	0,023798	0,023804

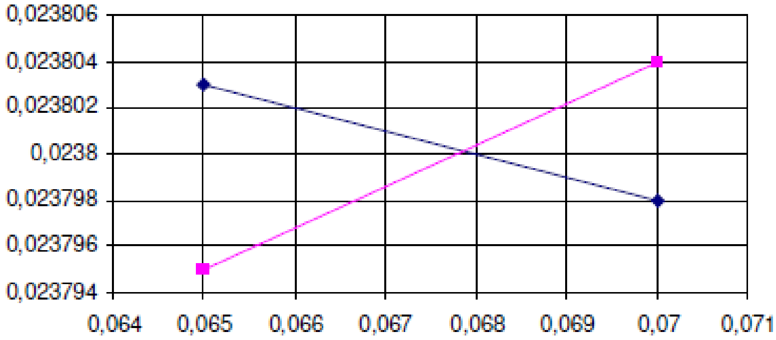

Fig. 7 Chart to determine CLF position

The intersection point corresponds to the position:

$X_{CLF} = 0.0679$ m

corresponding to 37.7% of the blade chord.

Deflection of a 1 m blade fragment at a torque of $M_X = 100$ Nm:

Y_N [m]	Y_S [m]
pkt 12952	pkt 437
0.03586	0.037842

Average deflection: z = 0.03586 m

Flexural stiffness in a plane perpendicular to the rotor plane (lower blade stiffness):

$$EJ_X = M_X / (2*z) = 1\ 394\ Nm^2$$

Deflection of a 1 m blade fragment at a torque of $M_Y = 100$ Nm:

X_N [m]	X_S [m]
pkt 12952	pkt 437
0.001104	0.001104

Average deflection: x = 0.001104 m

Flexural stiffness in the rotor plane (greater blade stiffness):

$$EJ_Y = M_Y / (2*z) = 45\ 290\ Nm^2$$

Deformation (twisting) of the 1 m blade section under twisting forces $M_Z = 100$ Nm:

$$\alpha = 0.016304 \text{ rad}$$

Torsional stiffness:

$$GJ_O = MZ/\alpha = 6\ 133 \text{ Nm/rad}$$

3.4. The end part

Fig. 8 shows the blade profile in the end section with two balancing tubes, divided into elements.

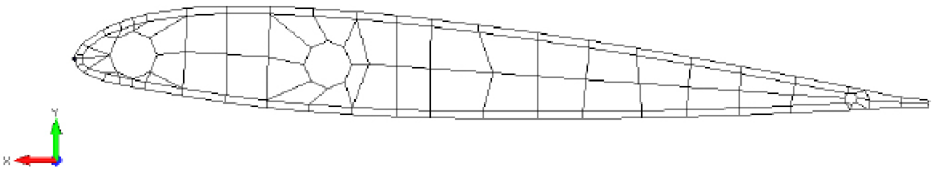

Fig. 8 The end section of a blade – model for CLF and stiffness location calculation.

Weight of the 1 m blade profile:

$$M_3 = 2.363 \text{ kg}$$

CG location:

$$X = -0.0625 \quad Y = -0.0013$$

Moments of inertia towards the coordinate origin:

$$I_{XX} = 0.7876 \quad I_{XY} = 0.000412$$
$$I_{YY} = 0.7999 \quad I_{YZ} = -0.00133$$
$$I_{ZZ} = 0.01246 \quad I_{ZX} = -0.07105$$

Moments of inertia towards CG:

$$I_{XX}= 0.1970 \quad I_{XY} = 0.000252$$
$$I_{YY} = 0.20069 \quad I_{YZ} = 1.345 \text{ E-13}$$
$$I_{ZZ} = 0.003914 \qquad I_{ZX} = 7.857 \text{ E-13}$$

CLF position:

Deflection calculated in two extreme points of the model profile of the blade (1 m length) under the force $F_Y = 100$ N

X [m]	Y_N [m]	Y_S [m]
	pkt 11850	pkt 564
0.065	-0.026496	-0.026469
0.070	-0.026491	-0.026480
0.075	-0.026486	-0.026490

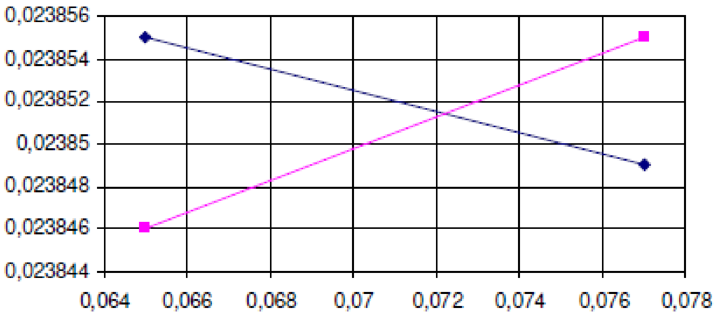

Fig. 7 Chart to determine CLF position

The intersection point corresponds to the position:

$X_{CLF} = 0.0722$ m

corresponding to 40.1% of the blade chord.

Deflection of a 1 m blade fragment at a torque of $M_X = 100$ Nm:

Y_N [m]	Y_S [m]
pkt 12952	pkt 437
0.038428	0.038428

Average deflection: z = 0.037 m

Flexural stiffness in a plane perpendicular to the rotor plane (lower blade stiffness):

$$EJ_X = M_X / (2*z) = 1\ 301\ Nm^2$$

Deflection of a 1 m blade fragment at a torque of $M_Y = 100$ Nm:

X_N [m]	X_S [m]
pkt 12952	pkt 437
0.0011119	0.0011119

Average deflection: x = 0.0009 m

Flexural stiffness in the rotor plane (greater blade stiffness):

$$EJ_Y = M_Y / (2*z) = 44\ 970\ Nm^2$$

Deformation (twisting) of the 1 m blade section under twisting forces $M_Z = 100$ Nm:

$$\alpha = 0.01719 \text{ rad}$$

Torsional stiffness:

$$GJ_O = MZ/\alpha = 5\,817 \text{ Nm}^2/\text{rad}$$

3.5. Moments of inertia of the whole blade

Moments of inertia calculated towards OY axis (upwards).

Radius is OZ axis, chord direction is OX (from trailing edge to the leading edge).

$I_{xx} = 41555 \quad I_{xy} = 0.00127$

$I_{yy} = 41595 \quad I_{yz} = -0.0165$

$I_{zz} = 0.0408 \quad I_{zx} = -0.9532$

4. Static deflection of the blade

For the validation of FEM model performed were calculations of the deflection of the blade fixed on the root with loads:

- At the blade end in a direction perpendicular to the profile chord,
- At the blade end in the direction along the profile chord,
- Under its own gravity.

The calculation results along with results of similar measurements for the blade are summarized in Table.

Tab. 1. Comparison of the three cases of loads.

No.	Load	Calculations		Measurement	
		Load N, Nm	Deform. mm	Load N, Nm	Deform. mm
1.	By force at the blade end in a direction perpendicular to the chord	50	299	50	177
2.	By force At the blade end in the direction along the chord	100	22.7	100	7
3.	By twisting forces	100		49	0.091
4.	By own gravity	-	164	-	172

5. BLADE VIBRATIONS

Table 2 lists types and frequency of blade vibrations, which were identified and calculated. The blade was modelled as fixed in the mounting holes.

Tab. 2. Basic forms of rotor blades vibrations

No.	Type of vibrations	Frequency Hz for n=0 rps
1.	I. Vertical bending	1.36
2.	I. Horizontal bending	6.42
3.	I. Vertical bending	8.72
4.	I. Horizontal bending	24.8
5.	I. Vertical bending	38.4
6.	I. Horizontal bending	48.9
7.	I. Vertical bending	80.8
8.	I. Horizontal bending	100.5
9.	I. Twisting	100.8

Fig. 9 shows an example of the type "III. vertical bending "

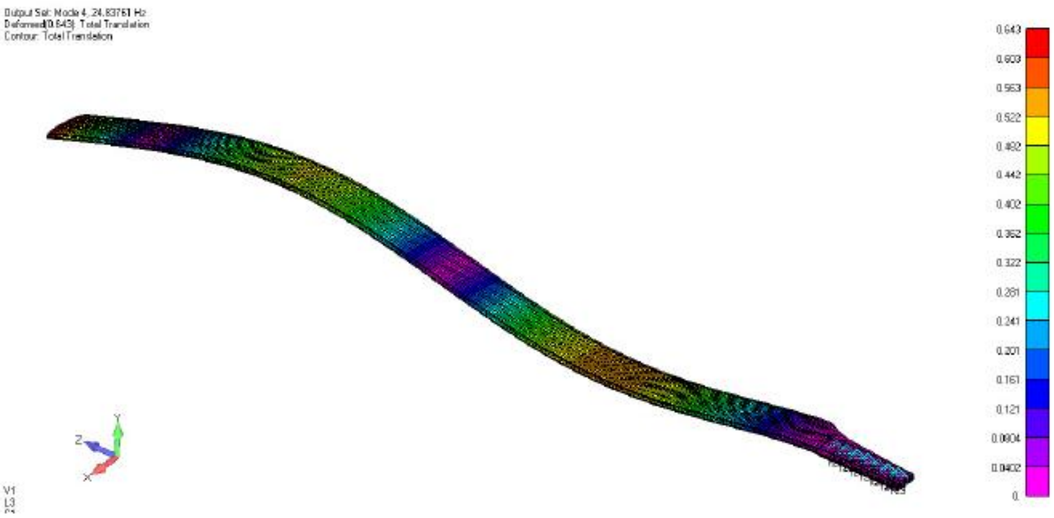

Fig.9 type "III. vertical bending " (example)

6. COMMENTS AND CONCLUSIONS

1. The blade is characterized by relatively large torsional and flexural stiffness

3. The calculations should be repeated after clarifying stiffness data of fixing (rotor head) or after introduction of significant structural changes (mass and stiffness data).